Reprinted from

Journal of Atmospheric and Terrestrial Physics, 1968, Vol. 30, pp. 213–226. Pergamon Press. Printed in Northern Ireland

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PERGAMON PRESS
OXFORD NEW YORK LONDON PARIS

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(*Received* 11 *August* 1967)

Abstract—The dispersion equation for a whistler mode propagation in a warm plasma, subjected to parallel static electric and magnetic fields, is derived by using a linearized coupled Vlasov–Maxwell equation.. From the derived dispersion equation, the amplitude constant α and phase constant β of the whistler mode are expressed in terms of static electric field E_0, static magnetic field B_0, the electron number density N_0, the electron temperature and the wave angular frequency ω. The effect of a weak static electric field on the propagation of a whistler mode is investigated in detail; the whistler mode may be amplified or attenuated according to whether $\mathbf{E}_0$ and $\mathbf{B}_0$ are in the same direction or in the opposite direction. The spatial rate of change of the wave amplitude and phase velocity of the whistler mode increase with $|E_0|$ in general. For a low-frequency wave propagation, α is directly proportional to E_0, ω, and N_0, and inversely proportional to B_0^3.

A whistler mode propagation in the magnetosphere is also considered. The results of study show that the effects of a static electric field on the propagation characteristic of a whistler mode are likely to be more important in the region of low geomagnetic latitude and high altitude rather than in the high latitude region.

1. Introduction

Most analyses of electromagnetic wave propagation in the ionospheric plasma in the past appear to have been concerned primarily with the effect of the earth's magnetic field (e.g. Ratcliffe, 1959, Ginzburg and Budden, 1961), but little or no attention has been given to the effect of a static electric field which may be present in the ionosphere. This is perhaps because, in general, the static electric field effects were presumed to be small. However the existence of a static electric field in the ionosphere or magnetosphere has been postulated by various workers in the studies of various ionospheric phenomena, for example, in the formation of F-region irregularities (Dagg, 1957; Alfvén, 1964; Willmore, 1966). It is also believed that the existence of turbulence in the magnetosphere must necessarily lead to the existence of a weak static electric field along the direction of the magnetostatic field in the steady state (Reid, 1965). Recently the experimental evidence of electrostatic field in auroral ionosphere has been reported by Mozer and Bruston (1967); e.g., the component of the d.c. electric field along the magnetic field direction had a magnitude as large as 20 mV/m. The importance of electric fields to magnetospheric and ionospheric phenomena appears to be well established by experiment and theory.

In view of the above, a natural question arises as to what, if any, is the effect of an electrostatic field on the propagating electromagnetic wave along the earth's magnetic field line in the ionospheric plasma. The answer to this question will be of interest since the electromagnetic wave under consideration might be of a man-made

radio signal used in a communication or used as a diagnostic tool for probing the plasma condition. It might also be of a radio noise of natural origin, such as whistler or v.l.f. emission, propagating in the ionosphere.

The whistler mode propagation through the ionospheric plasma has been discussed in detail by a number of workers (STOREY, 1953; RATCLIFFE, 1959; HELLIWELL and MORGAN, 1959; SMITH, 1961; SCARF, 1962; GALLET, 1964; HELLIWELL, 1965). On the other hand, the effect of a static electric field on the longitudinal propagation of circularly polarized waves in a finite temperature magneto-plasma has been investigated by this author. It is found that the presence of a transverse static electric field causes the cutoff frequency of the electromagnetic wave to shift (HSIEH, 1967*a*); in addition, it leads to a coupling of the longitudinal mode to a transverse circularly polarized mode (HSIEH, 1967*b*), whereas the presence of a longitudinal static electric field may significantly affect the amplitude and phase of the electromagnetic waves.

The purpose of this paper, therefore, is to discuss in detail the effect of a longitudinal static electric field upon the whistler mode propagation in the magneto-ionic medium, as defined by RATCLIFFE (1959). The thermal motion of an electron is considered although ion motion and collision effects are assumed to be negligible. The present discussion is based on a small-amplitude, one-dimensional analysis in which all time-varying quantities are assumed to have harmonic dependence of the form $\exp[j(\omega t - kz)]$, where ω and k are the wave angular frequency and propagation constant respectively. z and t denote, respectively, the spatial and time variables.

2. DISPERSION RELATION

Consider all quantities of interest to be composed of two parts; the time-independent part denoted by the subscripts '0', and the time-dependent part denoted by the subscript '1', e.g., the magnetic flux density $\mathbf{B}$ and electric field intensity $\mathbf{E}$ are written as

$$\mathbf{B} = \mathbf{B}_0(z) + \mathbf{B}_1(z, t) \text{ and } \mathbf{E} = \mathbf{E}_0(z) + \mathbf{E}_1(z, t)$$

and the electron distribution function f is written as

$$f = f_0(z, \mathbf{v}) + f_1(z, \mathbf{v}, t).$$

Suppose that the static electric field $\mathbf{E}_0$ and the magnetostatic field $\mathbf{B}_0$ are both directed along the positive z-direction. The dynamic electromagnetic fields in the electron gas are governed by Maxwell's field equation, which is expressed in the following manner:

$$E_{\pm} = \frac{-j\left(\frac{\omega e}{\varepsilon_0}\right)}{(\omega^2 - c^2k^2)} \int v_r e^{\pm j\varphi} f_1 \, d^3v,$$

$$E_{1z} = \frac{-je}{\omega\varepsilon_0} \int v_z f_1 \, d^3v, \tag{1}$$

where $c = 1/\sqrt{(\mu_0\varepsilon_0)}$ is the speed of light in free space, and μ_0 and ε_0 denote the permeability and dielectric constant of vacuum; $d^3v = (v_r \, dv_r \, d\varphi \, dv_z)$ is a volume

element in velocity space. On the other hand, the electron distribution is described by the Vlasov equation, which is written as

$$\left[j(\omega - kv_z) + \omega_z \frac{\partial}{\partial \varphi}\right] f_1 - a_z \frac{\partial f_1}{\partial v_z} = \frac{e}{m} M_-(f_0) E_- e^{j\varphi} + \frac{e}{m} M_+(f_0) E_+ e^{-j\varphi} + \frac{e}{m} E_{1z} \frac{\partial f_0}{\partial v_z}, \tag{2}$$

where

$$\begin{aligned} E_\mp &= \tfrac{1}{2}(E_{1x} \mp jE_{1y}), & B_\mp &= \tfrac{1}{2}(B_{1x} \mp jB_{1y}), \\ a_z &\equiv \left(\frac{eE_0}{m}\right), & \omega_z &= \left(\frac{eB_0}{m}\right), \\ M_\mp(f_0) &= \left[\left(1 - \frac{kv_z}{\omega}\right)\left(\frac{\partial f_0}{\partial v_r} \pm \frac{j}{v_r}\frac{\partial f_0}{\partial \varphi}\right) + \frac{kv_r}{\omega}\frac{\partial f_0}{\partial v_z}\right], \\ v_x &= v_r \cos\varphi, & v_y &= v_r \sin\varphi. \end{aligned} \tag{3}$$

Here E_- and E_+ denote the left and right-hand circularly polarized components of electric field respectively. E_{1z} is the longitudinal component of electric field. ω_z is the electron cyclotron frequency.

Suppose that the time-varying distribution function f_1 is written in the form

$$f_1(z, t, v_r, v_z, \varphi) = f_-(z, t, v_r, v_z)e^{j\varphi} + f_+(z, t, v_r, v_z)e^{-j\varphi} + g(z, t, v_r, v_z) \tag{4}$$

in which the first, second and third terms of the right-hand side can be regarded as the left-hand circularly polarized, the right-hand circularly polarized, and the longitudinal components of the distribution function respectively. Then in view of the fact that equation (2) must be valid for an arbitrary value of φ, the substitution of equation (4) into equation (2) yields a system of equations, expressing the function f_-, f_+ and g in terms of the electric field components E_-, E_+ and E_{1z} as follows:

$$\begin{aligned} j(\omega - kv_z + \omega_z)f_- - a_z \frac{\partial f_-}{\partial v_z} &= \frac{e}{m} M_-(f_0) E_-, \\ j(\omega - kv_z - \omega_z)f_+ - a_z \frac{\partial f_+}{\partial v_z} &= \frac{e}{m} M_+(f_0) E_+, \\ j(\omega - kv_z)g - a_z \frac{\partial g}{\partial v_z} &= \frac{e}{m}\frac{\partial f_0}{\partial v_z} E_{1z}. \end{aligned} \tag{5}$$

For the case where f_-, f_+ and g have v_z dependence of the form $e^{-\alpha_e v_z^2}$, in which $\alpha_e \equiv (m/2KT)$, they can be expressed explicitly in terms of E_-, E_+ and E_{1z} as follows:

$$f_\mp = \frac{\dfrac{e}{m} M_\mp(f_0) E_\mp}{j(\tilde{b} \pm \omega_z)} \quad \text{and} \quad g = \frac{\dfrac{e}{m}\dfrac{\partial f_0}{\partial v_z} E_{1z}}{j\tilde{b}}, \tag{6}$$

where

$$\tilde{b} = (\omega - \tilde{k}v_z), \qquad \tilde{k} = k + jK_1 \qquad \text{and} \qquad K_1 \equiv \left(\frac{eE_0}{KT}\right).$$

K and T denote the Boltzmann constant and the electronic temperature respectively.

Upon substituting equation (6) into equation (4) then combining with equation (1) the following set of equations is obtained:

$$1 + \frac{\pi\left(\frac{\omega e}{\varepsilon_0}\right)}{(\omega^2 - c^2k^2)} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{\frac{e}{m} M_{\mp}(f_0)}{(\tilde{b} \pm \omega_z)} v_r^2 \, dv_r \, dv_z = 0 \tag{7a}$$

and

$$1 + \frac{2\pi e}{\omega \varepsilon_0} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{e}{m} \frac{1}{\tilde{b}} \frac{\partial f_0}{\partial v_z} v_r v_z \, dv_r \, dv_z = 0. \tag{7b}$$

Equation (7b) is the dispersion relation for the longitudinal mode. Equation (7a) represents the dispersion relation for the transverse circularly polarized modes. The upper sign in this equation is taken for the left-hand circularly polarized mode and the lower sign is for the right-hand circularly polarized mode, which is of interest to the present investigation. Once the time-independent distribution function f_0 is known, the indicated integration in equation (7a) can be carried out. f_0, which must satisfy the time-independent part of the Vlasov equation for the case of a sufficiently weak static electric field in an electrically neutral electron gas, can be written in a Maxwellian form as follows:

$$f_0 = N_0 \left(\frac{\alpha_e}{\pi}\right)^{3/2} \exp\left[-\alpha_e(v_r^2 + v_z^2)\right], \tag{8}$$

where $\alpha_e = (m/2KT)$ and N_0 is the electron number density. Substitution of equation (8) into equation (7a) yields, for the right-hand circularly polarized mode, the following dispersion relation:

$$1 - \frac{c^2k^2}{\omega^2} = \frac{X}{(1 - Y)}\left[1 + \frac{V_1^2\tilde{k}^2}{2\omega^2} \frac{1}{(1 - Y)^2}\right], \tag{9}$$

where

$$X \equiv \left(\frac{\omega_p^2}{\omega^2}\right), \qquad Y \equiv \left(\frac{\omega_z}{\omega}\right), \qquad V_1 \equiv \sqrt{\left(\frac{2KT}{m}\right)},$$

$$\tilde{k} \equiv (k + jK_1), \qquad K_1 \equiv \left(\frac{eE_0}{KT}\right), \qquad \omega_z \equiv \left(\frac{eB_0}{m}\right),$$

$$\omega_p \equiv \sqrt{\left(\frac{e^2N_0}{m\varepsilon_0}\right)}.$$

It should be pointed out that the deriyation of equation (9) involves an evaluation of an integral of the form

$$G_0(x) \equiv \frac{j}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\alpha_e v_z^2}}{(v_z - \chi)} dv_z, \tag{10}$$

where $\chi = (1 - Y)\omega/\tilde{k}$.

This integral has been discussed in detail by STIX (1962) and his result is used here. When the term representing the LANDAU or cyclotron damping is neglected

and by taking only the first two terms of its asymptotic expansion, $G_0(\chi)$ can be given as

$$G_0(\chi) = \frac{-j}{\sqrt{(\alpha_e \chi)}} \left(1 + \frac{1}{2\alpha_e \chi^2}\right) \tag{11}$$

provided that $|\sqrt{\alpha_e}\chi|^4 \gg 1$ is satisfied. The case where $|\sqrt{\alpha_e}\chi| < 1$ has been discussed by SCARF (1962) in connection with the study of LANDAU damping and attenuation of whistlers.

It is of interest to note that when $V_1 = 0$, equation (9) is reduced to the familiar dispersion equation in the cold-plasma magneto-ionic theory.

3. PROPAGATION CONSTANTS

Equation (9) is a quadratic in k and can be conveniently rearranged as

$$Pk^2 + j2Sk - (Q + W) = 0, \tag{12}$$

where

$$P = \frac{c^2}{\omega^2}\left[1 - \frac{\omega^2}{c^2}\frac{2S}{K_1}\right],$$

$$W = 2K_1 S, \qquad S = \frac{K_1 V_1^2}{4\omega^2}\frac{X}{(1-Y)^3}$$

and

$$Q = 1 - \frac{X}{(1-Y)}. \tag{13}$$

For a real wave angular frequency ω, the factor P, Q, S, and W are real and equation (12) yields two complex roots which can be given as follows:

$$k = \pm\beta - j\alpha, \tag{14a}$$

where

$$\alpha = \frac{S}{P},$$

and

$$\beta = \sqrt{\left(\frac{(Q+W)}{P} - \alpha^2\right)}. \tag{14b}$$

In view of the fact that the time and spatial dependence is assumed to be in the form $\exp[j(\omega t - kz)]$, k given by equation (14a) with the upper sign corresponds to the propagation constant of the forward wave, and that with the lower sign corresponds to the propagation constant of the backward wave. Moreover a positive α represents the attenuation, while a negative α represents the amplification of the wave. Having determined the complex propagation constant k, the variation of the amplitude and phase of the wave with respect to various system parameters can be examined in detail. For convenience of discussion the amplitude coefficient (α/β_0) and phase coefficient (β/β_0), where $\beta_0 \equiv (\omega/c)$, are expressed as

$$\left(\frac{\alpha}{\beta_0}\right) = -\tfrac{1}{2}\delta Z \qquad \text{and} \qquad \left(\frac{\beta}{\beta_0}\right) = n_0 C, \tag{15a}$$

where

$$Z = \frac{r}{(1 - \frac{1}{2}\tau r)}, \qquad n_0 = \sqrt{\left(\frac{1+q}{1-\frac{1}{2}\tau r}\right)}$$

and

$$C = \left[1 - 2\frac{\delta^2}{\tau}\frac{r}{(1+q)}\left(1 + \frac{\tau}{8}Z\right)\right]^{1/2} \tag{15b}$$

in which

$$q \equiv \frac{X}{(Y-1)}, \qquad r \equiv \frac{X}{(Y-1)^3}, \qquad \tau \equiv \left(\frac{2KT}{mc^2}\right)$$

and

$$\delta \equiv \left(\frac{eE_0}{mc\omega}\right). \tag{15c}$$

It should be noted that the amplitude coefficient (α/β_0) is proportional to δ, which in turn is proportional to E_0. For a whistler mode of interest, $(\tau r)/2 < 1$ so that $Z > 0$. Consequently the amplitude constant α can be either positive or negative according to whether $E_0 < 0$ or $E_0 > 0$. In view of the fact that the direction of $\mathbf{B}_0$ is taken in the positive z-direction, which is also that of the wave propagation, $E_0 < 0$ means that the electrostatic field is directed in the direction opposite to the wave vector, while $E_0 > 0$ signifies that $\mathbf{E}_0$ is in the direction of the wave vector. For the case $\alpha > 0$ (i.e., $E_0 < 0$) the whistler mode under consideration suffers an attenuation, whereas for the case $\alpha < 0$ (i.e., $E_0 > 0$) it experiences an amplification. The attenuation corresponds to the absorption of whistler mode electromagnetic wave energy by the plasma electrons, while the amplification corresponds to the situation of extraction of energy from the plasma.

On the other hand it should be noted that the phase coefficient (β/β_0) is nothing but the refractive index (c/v_0), with v_0 being the phase velocity of the wave, and n_0 representing the refractive index for the case of zero-static electric field, while the factor C represents the correction factor due to the presence of the static electric field. Thus the effects of E_0 upon the propagation of the whistler mode under consideration are characterized by equation (15a). The numerical illustration of equation (15a) is shown in Figs. 1 and 2 for a conveniently chosen set of parameters. The variations of the amplitude coefficient (α/β_0), and the phase coefficients (β/β_0) with the parameters X and Y for a fixed value of δ and τ, are illustrated in Figs. 1(a) and 1(b) respectively. It is observed from Fig. 1(a) that (α/β_0) increases monotonically with X when Y is fixed, whereas it decreases as Y increases when X is fixed. Moreover the rate of increase of (α/β_0) with respect to X decreases as Y increases. Thus, Fig. 1(a) suggests that the spatial rate of change of the wave amplitude increases with the electron number density N_0 when the wave angular frequency ω and magnetostatic field B_0 are fixed, whereas it decreases with an increase in the strength of the magnetostatic field when N_0 is fixed.

Figure 1(b) indicates that the phase coefficient (β/β_0) increases with X when Y is fixed, whereas it decreases as Y increases when X is fixed. Thus, Fig. 1(b) suggests that the phase velocity of the whistler mode under consideration decreases as the

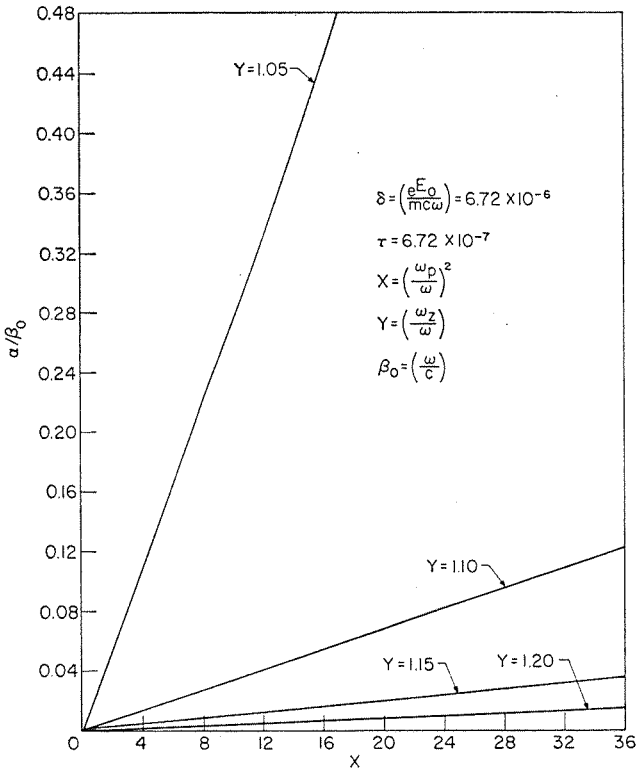

Fig. 1(a). The plot of (α/β_0) vs. X with Y as parameter for $\delta = 6{\cdot}72 \times 10^{-6}$ and $\tau = 0{\cdot}672 \times 10^{-6}$ (or $T = 2000°K$)

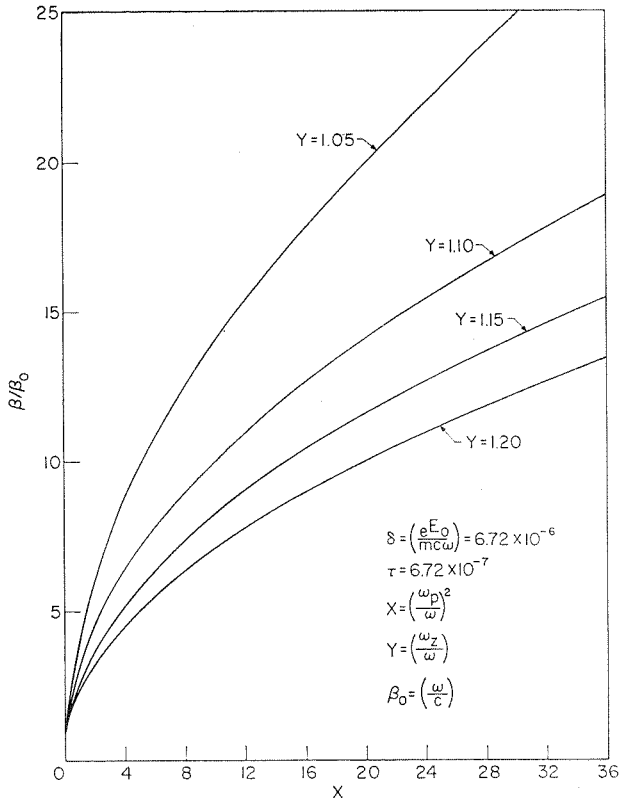

Fig. 1(b). The plot of (β/β_0) vs. X with Y as parameter for $\delta = 6{\cdot}72 \times 10^{-6}$ and $\tau = 0{\cdot}672 \times 10^{-6}$ (or $T = 2000°K$).

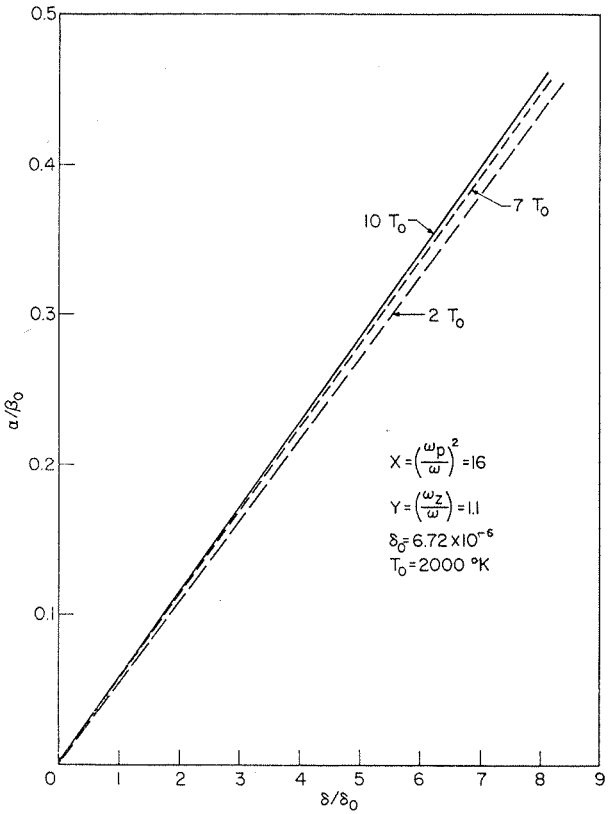

Fig. 2(a). The plot of (α/β_0) vs. δ with τ as parameter for $X = 16{\cdot}0$ and $Y = 1{\cdot}1$.

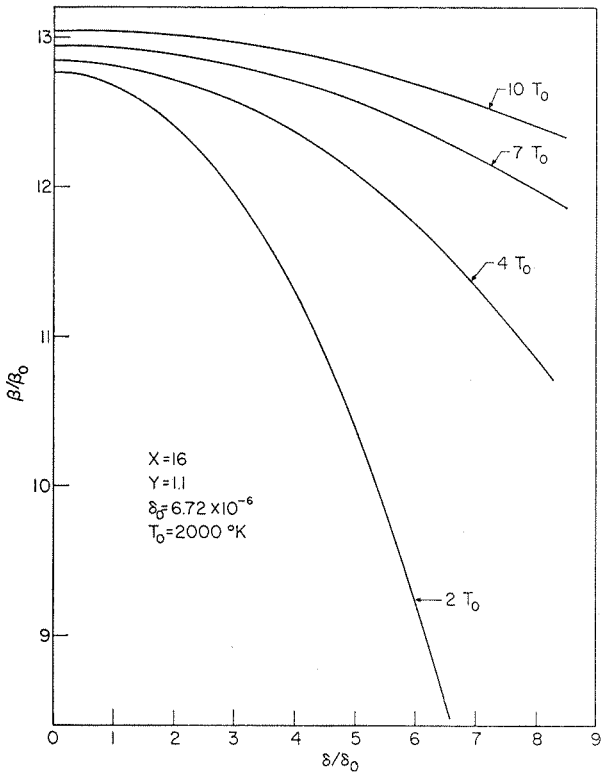

Fig. 2(b). The plot of (β/β_0) vs. δ with τ as parameter for $X = 16{\cdot}0$ and $Y = 1{\cdot}1$.

electron number density N_0 increases when ω and B_0 are fixed, whereas it increases as B_0 increases when N_0 is fixed.

On the other hand, the variation of (α/β_0) and (β/β_0) with the parameters δ and τ for a fixed value of X and Y is shown in Figs. 2(a) and 2(b) respectively. It is observed from Fig. 2(a) that (α/β_0) is directly proportional to the parameter δ and it increases with τ when δ is fixed. Thus, Fig. 2(a) suggests that the spatial rate of change of the wave amplitude increases proportionally with E_0 when N_0, B_0, ω and τ are fixed. Figure 2(b) indicates that (β/β_0) decreases with δ when τ is fixed, whereas it increases with τ when δ is fixed. Figure 2(b) thus suggests that for a given value of N_0, B_0, and ω, the phase velocity of the whistler mode increases with $|E_0|$ when T is fixed, whereas it decreases as T increases when E_0 is fixed.

Figures 2(a) and 2(b) together tend to suggest that the presence of an electrostatic field in the system may modify both the amplitude and phase of the whistler mode under consideration significantly. On the other hand, a change in the electron temperature T appears to have only little effect upon the amplitude, but significant effect on the phase of the whistler mode.

It is of interest to note that when the parameters X, Y, and τ are such that

$$1 \ll Y \ll X \qquad \text{and} \qquad |X/Y^3| \ll 1/\tau, \tag{16}$$

equations (15) yield

$$(\alpha/\beta_0) = -\tfrac{1}{2}\delta\sigma^2/Y \qquad \text{and} \qquad (\beta/\beta_0) = \sigma\sqrt{\left(Y^2 - \frac{2\delta^2}{\tau}\right)}, \tag{17}$$

where $\sigma \equiv (\omega_p/\omega_z)$. Equation (17) can also be written as

$$\alpha = -\frac{1}{2}\left(\frac{E_0}{cB_0}\right)\left(\frac{\omega_p^2}{\omega_z^2}\right)\left(\frac{\omega}{c}\right) \tag{18a}$$

and

$$\left(\frac{v_0}{c}\right) = \sqrt{\frac{\omega_z\omega}{\omega_p^2}}\,\frac{1}{\sqrt{1 - \left(\frac{m}{KT}\right)\left(\frac{E_0}{B_0}\right)^2}} \tag{18b}$$

which suggests that, for the low frequency whistler mode propagation, the spatial rate of change of wave amplitude α is directly proportional to E_0, N_0, and ω, and inversely proportional to (B_0^3). In other words, for larger N_0, there will be more electrons available for participating in the exchange of energy with the electromagnetic wave. On the other hand, the phase velocity v_0 of the whistler mode is proportional to $\sqrt{(\omega)}$ and it increases as E_0 increases regardless of the algebraic sign of E_0, whereas it decreases as the electron temperature T increases.

4. 'Whistler Mode' Propagation in Magnetosphere

A class of the electromagnetic wave of natural origin known as 'whistler' and 'v.l.f. emissions' can propagate through the ionosphere and magnetosphere along the geomagnetic field line in the whistler mode (Helliwell, 1965) when the wave frequency ω is in the proper range such that $\omega < \omega_z < \omega_p$.

The magnetosphere can be regarded as consisting of a neutral (hydrogen) plasma imbedded in the geomagnetic field. The background plasma density varies in space

and time and the field is approximately a dipole subject to small perturbation and the distortion of the solar wind. Above the ionosphere (e.g., 1000 km) the medium is slowly varying electromagnetically in the sense that the change in number density N_0 is small in the space of a wavelength (RATCLIFFE, 1959). Therefore, the propagation characteristics of the uniform medium which are given in the preceding section are applicable locally in the magnetosphere under a quiescent condition. If the path integral of the amplitude constant α is assumed to describe the net growth or decay of wave amplitude, then the amplification or absorption of power in a wave which propagates through a slowly varying medium is given by

$$A(\mathrm{dB}) = -10 \log_{10} \left[\exp \left\{ \int_{s_1,\ \mathrm{path}}^{s_2} 2\alpha(\omega, s)\, ds \right\} \right], \tag{19}$$

where α is the local amplitude exponent defined by equations (15a) or (18a), and $s_1 \leqq s \leqq s_2$ specifies the region where $\mathbf{E}_0$ and $\mathbf{B}_0$ are parallel and directed along the direction of wave propagation. ds is a differential path length along the geomagnetic field line. It should be noted that α depends on the quantities ω_z, ω_p, T and E_0, which are in turn dependent on position along the path in general. To evaluate the integral in equation (19), therefore, a knowledge of the variation of system parameters ω_z, ω_p, T, and E_0 is required.

If the geomagnetic field is approximated by a pure dipole field, then the whistler path is defined by the equation for a field line

$$\frac{R}{R_0} = \frac{\cos\theta}{\cos\theta_0}, \tag{20}$$

where (R/R_0) is the radial distance in earth radii, θ is the geomagnetic latitude, θ_0 is the latitude of the path at the earth's surface and R_0 is the radius of the earth. The electron cyclotron frequency along this path has the form

$$\omega_z = \omega_{z0} \frac{(1 + 3\sin^2\theta)^{1/2}}{\cos^6\theta}, \tag{21}$$

where ω_{z0} is its equatorial value ($= 5{\cdot}50 \times 10^6 \cos^6\theta_0$ rad./sec).

As for the electron plasma frequency variation along this path the so-called "gyro-frequency model" has been used by several authors (STOREY, 1953; GALLET, 1959; SMITH, 1961; DOWDEN, 1961) in the study of whistlers. In this model it is assumed that the electron density tends to be proportional to the strength of the earth's magnetic field, i.e.,

$$\omega_p^2 = \kappa\omega_z, \tag{22}$$

where κ is a constant which, in general, may depend on θ_0. Equation (22) represents a good empirical model for $45° \leqq \theta_0 \leqq 65°$.

On the other hand the electron temperature T within the magnetosphere is not uniform in general (CHAPMAN, 1960) so that T varies along the whistler path. However for most whistler analyses T has been regarded as constant along the path.

With regard to the strength and variation of the electrostatic field E_0 along the geomagnetic field line, it appears that no experimentally observed data has ever been reported in the literature. In his theoretical estimate REID (1965) showed that if the

source level at which the electrostatic potential originates were taken at the equatorial plane, which lies 42560 km above the surface of the earth, measured along the field line originating at 65° geomagnetic latitude, then a longitudinal electrostatic field will be about 5×10^{-7} V/m throughout the magnetosphere. However this estimate does involve a certain amount of guesswork as to the magnitude of source potential. Thus using equations (15) and (20), and assuming a uniform temperature and electrostatic field along the path in the magnetosphere, the absorption or amplification A, given by equation (19), can in principle be evaluated. However, for an illustration, it is of interest to consider the case in which the wave angular frequency ω is much smaller than the minimum gyrofrequency along the path, ω_{z0}, i.e., $\omega \ll \omega_{z0}$. If the electron temperature T in the magnetosphere is assumed to be in the range between 2000°K and 20,000°K, then the value of (V_1/c) will be in the range between $0{\cdot}82 \times 10^{-3}$ and $2{\cdot}57 \times 10^{-3}$. Since along the whistler path $\omega_z < \omega_p$, it is not difficult to see that the inequalities (16) are satisfied, so that equations (18a) and (18b) are applicable to the analysis of the low-frequency components of a whistler or v.l.f. emission in the magnetosphere. Consequently, with the aid of equations (21) and (22), equations (18a) and (18b) become respectively

$$\alpha = -\,0{\cdot}975 \times 10^{-6}\left(\frac{E_0 \omega \kappa}{{\omega_{z0}}^2}\right)\psi(\theta), \text{ nepers/m} \tag{23a}$$

$$\frac{v_0}{c} = \sqrt{\frac{\omega}{\kappa}}\left[1 - \left(4{\cdot}52 \times 10^7 \frac{E_0}{T\omega_{z0}}\right)^2 \psi(\theta)\right]^{-1/2}, \tag{23b}$$

where

$$\psi(\theta) = \frac{\cos^{12}\theta}{(1 + 3\sin^2\theta)}. \tag{23c}$$

The plot of $\psi(\theta)$ vs. θ (in Fig. 3a) shows that the function $\psi(\theta)$ decreases monotonically with the geomagnetic latitude θ from $\psi(0) = 1$ to $\psi(45°) = 0{\cdot}01$. The value of $\psi(\theta)$ for $\theta > 45°$ is negligibly small compared to $\psi(0)$. Consequently equation (23a) suggests that the amplitude constant α along the whistler path under consideration has a larger value in the lower latitude region, near the equatorial zone, than in the high latitude region.

On the other hand the absorption or amplification along the path, from the equatorial plane to a height of h km above the earth's surface, can be given by equation 19 as

$$A(d\text{B}) = 54\left(\frac{E_0 \omega \kappa}{{\omega_{z0}}^2}\right)\Psi(\theta_1) \tag{24a}$$

where

$$\Psi(\theta_1) = \int_0^{\theta_1} \frac{\cos^{13}\theta}{(1 + 3\sin^2\theta)^{1/2}}\, d\theta \tag{24b}$$

in which θ_1 is given by

$$\frac{\cos\theta_0}{R_0} = \frac{\cos\theta_1}{(R_0 + h)}. \tag{24c}$$

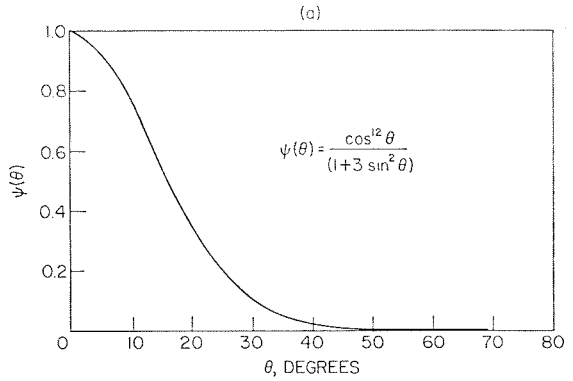

Fig. 3(a). The plot of $\psi(\theta)$ vs. θ.

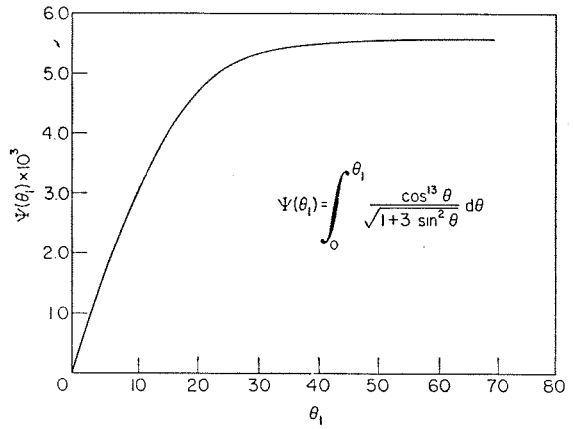

Fig. 3(b). The plot of $\Psi(\theta_1)$ vs. θ_1.

The plot of $\Psi(\theta_1)$ vs. θ_1 (in Fig. 3(b)) shows that the function $\Psi(\theta_1)$ increases monotonically from 0 at $\theta_1 = 0$ to $5{\cdot}5 \times 10^{-3}$ at $\theta_1 = 45°$, then $\Psi(\theta_1)$ remains constant for $\theta_1 > 45°$.

It should be noted that there is a path which has frequently been considered by various workers (e.g., SMITH, 1961; BRICE, 1964; LIEMOHN, 1967) in the study of whistlers. It is the path specified by $\theta_0 = 60°$ or $R(\theta = 0) = 4R_0$, which gives $\omega_{z0} = 8{\cdot}6 \times 10^4$ rad./sec or $f_z = 13{\cdot}7$ kc/sec. Along this path the value of κ, which appeared in equation 22, can be taken as $\kappa = 2\pi \times 10^6$ rad./sec. Then for the frequency range $f \leqq 1$ kc/sec, equations (23a) and (24) can be used. For an illustration, suppose that the electrostatic field along the path is taken as $E_0 = -5 \times 10^{-7}$ V/m; then a whistler mode electromagnetic wave with a frequency of one kc/sec will suffer an attenuation of $2{\cdot}64 \times 10^{-12}$ nepers/m at most. In this case the absorption for $\theta_1 = 45°$ will be $8{\cdot}03 \times 10^{-7}$ dB which is negligibly small. However, when a path with larger values of θ_0 and E_0 is considered, both α and A can be increased considerably.

5. CONCLUDING REMARKS

In considering a whistler mode propagation an attempt has been made to take into account the effect of a weak static electric field which might be present in a

warm magneto-ionic medium. The equilibrium distribution function of the electron, f_0, is assumed to be Maxwellian. Furthermore, it is also assumed that the static electric field E_0 in the direction of the magnetostatic field $\mathbf{B}_0$ is sufficiently weak so that the drift motion in the direction of wave propagation is small and does not modify the distribution function f_0 significantly. The drift velocity of electrons due to $\mathbf{E}_0$ is assumed to be negligibly small in comparison to the wave phase velocity. Thus the medium under consideration is regarded as essentially stationary under a weak static electric field.

The effect of the presence of electrostatic field on a whistler mode electromagnetic wave propagation is characterized by equation (15). The method of analysis discussed in Section 3 is most likely to be adequate for consideration of a whistler mode propagation in the magnetosphere where the collision effect is usually regarded as negligible. The application of the theory is illustrated in Section 4 for a low frequency wave. The effects of electrostatic field E_0 on the propagation of these low frequency waves is characterized by equation (23), which suggests that the electrostatic field effect on the amplitude and phase velocity of the whistler mode will likely be more significant in the region of low geomagnetic latitude and high altitude rather than in the high latitude region. In view of the lack of knowledge regarding the strength and variation of the electrostatic field in the magnetosphere, a reasonably accurate numerical estimation of the value of α or A is not possible at this time. However, the method illustrated in Section 4 for estimating the value of α or A can be used profitably when more data regarding E_0 become available in the future. Finally, it should be pointed out that the method of analysis developed in this paper can be extended to include such effects as ion-motion, collisions, and the drift motion of a plasma particle under the influence of a strong electrostatic field. If this is done then it can also be used for the consideration of electrostatic field effects on a whistler mode propagation in the ionosphere, particularly in the F-region where collision effects may be significant and the electrostatic fields are believed to be present.

Acknowledgement—The work was supported by the National Aeronautics and Space Administration under Grant NsG 696.

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